Unbiasedness of generalized score functions and sequential structures of expectations

H. Matsuzoe[1], A.M. Scarfone[2], T. Wada[3]

[1]Nagoya Institute of Technology

[2]Istituto dei Sistemi Complessi (ISC-CNR) c/o, Politecnico di Torino

[3]Ibaraki University

Information geometry is one of differential geometrical methods for mathematical sciences. In Riemannian geometry, a pair of a Riemannian metric and its Levi-Civita connection is important, whereas a triplet of a Riemannian metric and a pair of dual affine connections is important in information geometry. In the case of an exponential family, such as the set of all Gaussian distributions, we can naturally obtain a pair of dually flat affine connections, and this pair of affine connections has essential roles in geometric theory of statistical inferences. In particular, it is known that the dually flat structure can be obtained from the unbiasedness of score functions. A maximum likelihood method is characterized by a geodesic projection with respect to the dually flat structure, and a Legendre transformation structure is also obtained from this dually flat structure.

In the theory of nonextensive statistical physics, long-tailed probability distributions are also useful. For long-tailed probability distributions, the standard expectation does not exist in general. Therefore, the notion of escort distribution has been introduced, and the expectation with respect to the escort distribution has been studied. An escort distribution gives a suitable weight for tail probability, and the weight is characterized by an unbiasedness of generalized score functions.

In this presentation, we consider a sequential structure of escort expectations on a deformed exponential family. A deformed exponential family is a generalization of exponential families, which includes important classes of long-tailed probability distributions. It is known that a deformed exponential family naturally has at least three kind of different Riemannian metrics endowed with dual affine connections. In other words, we obtain three kind of different statistical manifold structures. We can find that such statistical manifold structures are obtained from a sequential structure of escort expectations. In particular, in the case of a q-deformed exponential family, the Fisher metric can be obtained from the second escort expectation. In this presentation, after reviewing the q-exponential case, we will consider the kappa-exponential case. Then we will obtain a new divergence function for a kappa-exponential family.

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